

## In the name of Allah

Slide Attacks with Probability 1  
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- Slide cryptanalysis of a blowfish variant (with success prob of 1)
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## Slide Cryptanalysis

- E is a block cipher.  $E_k$  the encryption function with key k

- **Preconditions of slide attack:**

1.  $E_k$  can be written as the composition of several identical functions

$$E_k(P) = (F_{K'} \circ F_{K'} \circ \dots \circ F_{K'})(P)$$

2. having 2 known-plaintext equations

$$F_{K'}(x_1) = y_1$$

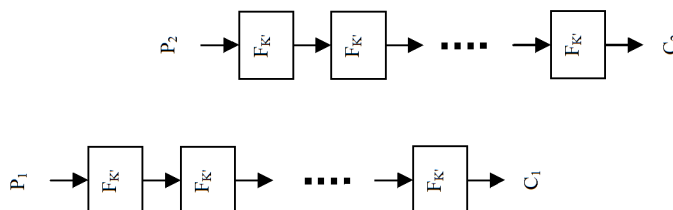
$$F_{K'}(x_2) = y_2$$

the key  $k'$  can be “*easily*” recovered

## Slide Cryptanalysis

- **Slide attack :**

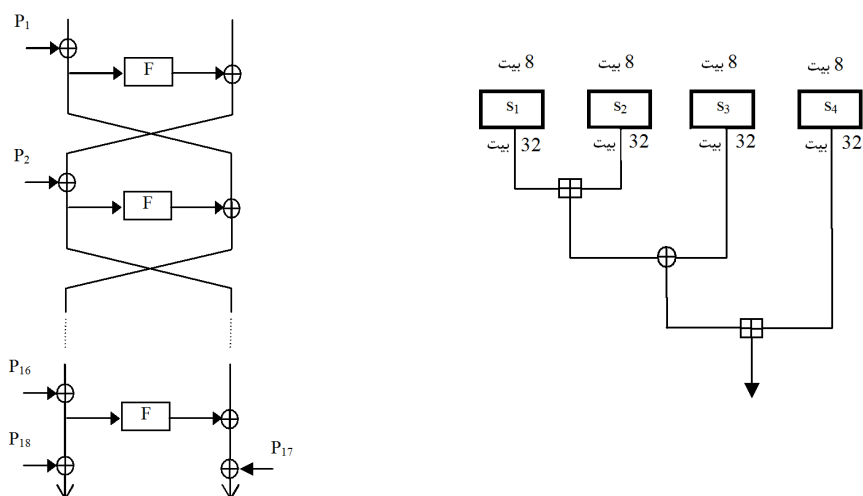
1. Look for a pair of plaintexts ( $P_1, P_2$ ) st  $F_{K'}(P_1) = P_2$   
 $\rightarrow F_{K'}(C_1) = C_2$



## Slide Cryptanalysis

- **Slide attack (cont'd) :**
  2.  $F_K'(P_1)=P_2$  &  $F_K'(C_1)=C_2$  are expected to reveal  $k'$ .
- How to find  $(P_1, P_2)$  st  $F_K'(P_1)=P_2$  ?
  - Common approach: gather almost  $2^{n/2}$  known pairs  $(P_i, C_i)$ .
  - Birthday paradox  $\rightarrow$  with good prob, there is at least 1 pair  $(P_1, P_2)$  st  $F_K'(P_1)=P_2$
  - The existence of such pairs is probabilistic.
- Our method guarantees the existence of such a pair with prob=1, for some special cases.

## Blowfish



### Slide cryptanalysis of Blowfish – Pi's [BirWag]

- Consider Blowfish – Pi's
- Consider the eq  $F(x)=y$

$$[(s_1(x_1) + s_2(x_2)) \oplus s_3(x_3)] + s_4(x_4) = y$$

- This eq gives 32 bits of info about the sbox entries.
- Every slide pair  $\rightarrow 2 \times 32 = 64$  bits of info.
- 1024 sbox entries in total .  $2^9 = 512$  slide pair needed.

### Slide cryptanalysis of Blowfish – Pi's [BirWag]

- Consider Blowfish – Pi's
- How to find a slide pair?
- Consider the 2 sets  $\{X_i \mid i=1,2,\dots,2^{16}\}$  and  $\{Y_j \mid j=1,2,\dots,2^{16}\}$
- Birthday paradox  $\rightarrow$  with prob  $\sim 0.5$  , there is  $X_i$  &  $Y_j$  st  $F(R) \oplus X_i = Y_j$
- $\rightarrow F_1(R, X_i) = (Y_j, R)$  : a slide pair.

### Slide cryptanalysis of Blowfish – Pi's

- **Consider Blowfish – Pi's**
- The previous approach for obtaining a slide pair is probabilistic.
- How to find a slide pair with **prob 1**?
- The idea is to use chosen plaintexts instead of known plaintexts.
- $(R, X_i)$  &  $(Y_j, R)$  are a slide pair iff  $F(R) \oplus X_i = Y_j$   
or  $X_i \oplus Y_j = F(R)$
- $F(R)$  is constant, so if we choose  $X_i$  &  $Y_j$  st  $X_i \oplus Y_j$  spans all 32bit values, one will equal  $F(R)$ .

### Slide cryptanalysis of Blowfish – Pi's

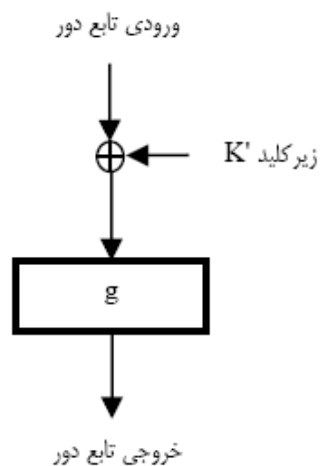
- **Consider Blowfish – Pi's**
- One such set of texts is  $X_i = a_i || 0_{16}$  and  $Y_i = 0_{16} || a_j$  where  $a_i = i$ .  $X_i \oplus Y_j = a_i || a_j$
- When  $i$  &  $j$  span the set  $\{1, 2, \dots, 2^{16}\}$  independently,  $a_i || a_j$  will span the set  $\{1, 2, \dots, 2^{32}\}$
- So for one  $(i, j)$  :  $X_i \oplus Y_j = F(R)$
- NOTE : we are sure there is one slide pair, but we can't tell it from other pairs.
- the process of identifying the slide pair is as before.

## Slide cryptanalysis of Blowfish – Pi's

- **Consider Blowfish – Pi's**
- for each R there is one slide pair with certainty.
- we need 512 slide pairs
  - change R and each time use the above-mentioned type of texts.

## extension to nonFeistel algorithms

- $g$ 
  - arbitrary
  - unKeyed
  - invertible
- $(p, p')$  is a slide pair iff
  - $p' = g(p \oplus K')$
  - or  $g^{-1}(p') \oplus p = K'$



### extension to nonFeistel algorithms

- $2n =$  block lengths
- put  $p_i = a_i || 0_n$  and  $g^{-1}(p'_j) = 0_n || a_j$  where  $a_i = i$  for  $i = 0, 1, \dots, 2^n - 1$   $g^{-1}(p'_j) \oplus p_i = a_i || a_j$
- When  $i$  &  $j$  span the set  $\{1, 2, \dots, 2^n\}$  independently,  $a_i || a_j$  will span the set  $\{1, 2, \dots, 2^{2n}\}$
- so  $p_i = a_i || 0_n$  and  $p'_j = g(0_n || a_j)$
- Data complexity is exactly  $2^n + 2^n$

### extension to nonFeistel algorithms

- the key combining operation is not restricted to simple xor.
- e.g. for  $p = (p_1, p_2, p_3, p_4)$  and  $K' = (k'_1, k'_2, k'_3, k'_4)$ 
  - $p + K' = ((p_1 + k'_1) \bmod 2^{32}, (p_2 + k'_2) \bmod 2^{32}, (p_3 + k'_3) \bmod 2^{32}, (p_4 + k'_4) \bmod 2^{32})$
- $(p, p')$  is a slide pair iff  $p' = g(p + K')$   
or  $g^{-1}(p') - p = K'$

### extension to nonFeistel algorithms

- put 
$$p_{i,j} = (-a_i, -a_j, 0_{32}, 0_{32})$$
$$p'_{m,n} = g(0_{32}, 0_{32}, a_m, a_n)$$

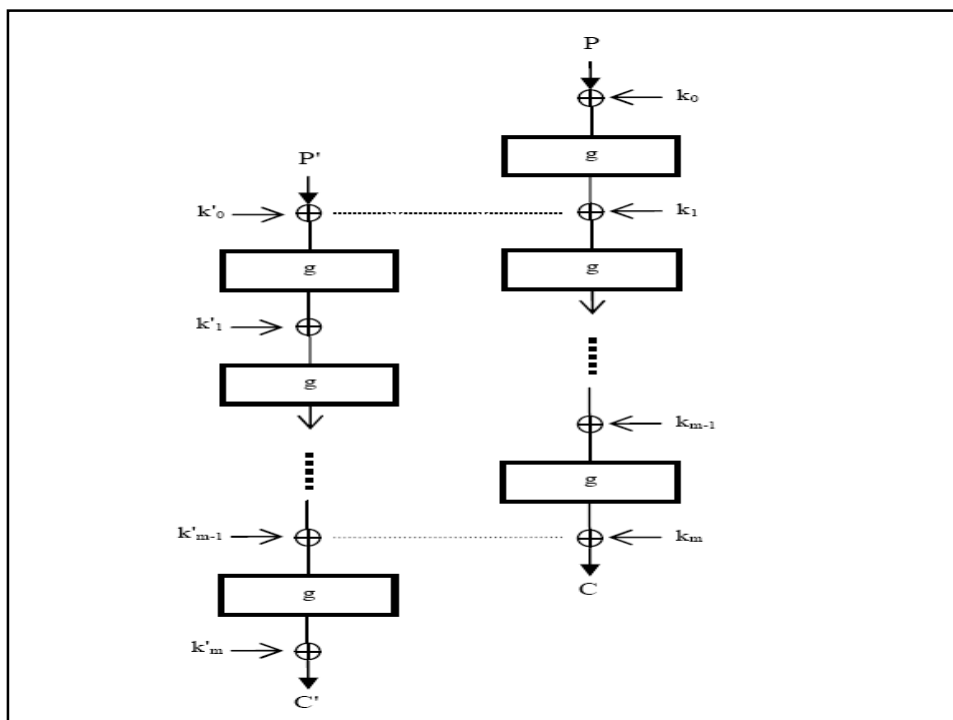
- where  $a_i=i$  for  $i=0, 1, \dots, 2^{32}-1$

$$g^{-1}(p'_{m,n}) - p_{i,j} = (a_i, a_j, a_m, a_n)$$

### Biham's related-key attack

- Applicable to algorithms whose
  - $i$ -th rnd func is the same as the 1<sup>st</sup> rnd func
  - $(i+1)$ -th rnd func is the same as the 2<sup>nd</sup> rnd func
  - ...
 for some small  $i$  (1 or 2)
  - **plus** for each key  $K$  with subkeys  $k_0, k_1, \dots$  there is a related-key  $K'$  with subkeys  $k'_0, k'_1, \dots$  st  $k'_0=k_i, k'_1=k_{i+1}, \dots$





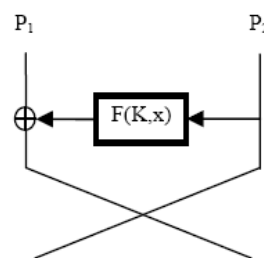
### Biham's related-key attack

- If  $P' = g(P \oplus k_0)$  then  $C' = g(C) \oplus k'_m$  and these two relations are expected to reveal info about subkeys.
- The problem is to find  $(P, P')$  st  $P' = g(P \oplus k_0)$
- Traditional method : appeal to birthday paradox
  - probabilistic in nature
- Our method can guarantee the existence of  $(P, P')$  with prob 1

### Biham's related-key attack

- 1- Feistel type rnd func
  - $F$  : absolutely arbitrary
- We have to find  $(P_1, P_2)$  and  $(P'_1, P'_2)$  st
 
$$P'_1 = P_2$$

$$F(K, P_2) \oplus P_1 = P'_2$$



### Biham's related-key attack

- 1- Feistel type rnd func

- put
 
$$P_1^{(i)} = a_i || 0_n$$

$$P_2^{(j)} = 0_n || a_j$$

then  $P_1^{(i)} \oplus P_2^{(j)} = a_i || a_j$

and for each constant  $R$ ,  $F(K, R) = P_1^{(i)} \oplus P_2^{(j)}$  occurs for some  $i$  &  $j$ .

### Biham's related-key attack

- 1- Feistel type rnd func

- So a possible candidate is

$$(P_1^{(i)}, P_2) = (a_i || 0_n, R)$$

$$(P_1', P_2^{(j)}) = (R, 0_n || a_j)$$

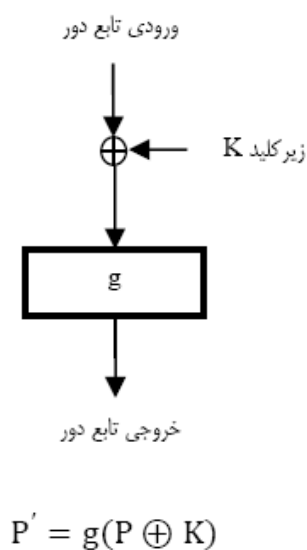
where  $a_i = i$  for  $i = 0, 1, \dots, 2^n - 1$  ( $2^n =$  block len)

- the combining operation need NOT be restricted to xor.

### Biham's related-key attack

- 1- nonFeistel type rnd func

- $g$  as before (SPN a special case)
- the key combining op need NOT be restricted to xor.
- previous solution works



### Biham's related-key attack

- **Conclusion** : The idea works well for  $i=1$  and with some restriction on the round structure.
- Avenue for future research :
  - find similar methods for  $i=2$  .

### Biham's related-key attack

Any Question?